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LETTER TO THE EDITOR

Longitudinal field muon spin relaxation: a new probe of two-dimensional electron systems

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Abstract. It is shown that longitudinal field muon spin relaxation may be used to probe the local charge density correlations in two-dimensional systems and heterostructures. Magnetic quantum oscillations in the local density of extended states show up in the relaxation time T_1. This method should reveal the variation of electronic parameters within the sample.

Muons are a well known and valuable probe of their local environment, and are commonly used as such in solid state physics [1]. Typically the muon is trapped on some (set of) sites in the solid, and the way in which the muon spin relaxes is a sensitive probe of the magnetic field arising in the immediate vicinity of the muon.

Rather less understood but equally interesting is the way in which muons may diffuse within the solid. This motion is governed by the coupling of the muon to the low energy excitations in the system, and observation of it allows experimental testing of some rather fundamental theoretical work on quantum diffusion [2–4].

In this letter we show that the physics of this second phenomenon may be used to explore the local properties of two-dimensional films and heterostructures. Such measurements are rather rare—in fact the only such local probe so far employed (to our knowledge) in two-dimensional systems has been nuclear magnetic resonance (NMR) in a recent experiment by Berg *et al* [5], whose results were in remarkable agreement with the theoretical predictions of Vagner and Maniv [6]. These experiments probed the local electron spin correlations in a GaAs heterostructure.

Our idea is that longitudinal field muon spin relaxation (LF-μSR) experiments be attempted on similar heterostructures, and that these will reveal the local charge correlations in the system. They will also allow a rather interesting variation (through the varying longitudinal field) of the dimensionless coupling parameter K between the muons and the background electrons; this parameter controls the muon diffusion rate.

Now in a typical LF-μSR experiment, one measures the decaying positron time spectrum of individual muons, which as a function of time and angle has the form [1]

$$N(\theta,t) = N_0 \exp(-t/\tau_\mu)[1 + AG_z(t)\cos\theta] \tag{1}$$

where τ_μ is the muon life time (2.2 μs), A is the asymmetry parameter and the relaxation function $G_z(t)$ has been discussed by various authors in different limiting cases [7–10]. In this letter we shall be interested in the limit described by the random fluctuating field approximation [8], which essentially involves two parameters ν and δ.

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The hopping rate ν is the inverse of the correlation time τ_c describing the fluctuating random field, and $m_\mu^2\delta^2/e^2 = \langle H_x^2\rangle = \langle H_y^2\rangle = \langle H_z^2\rangle$ is the second moment of the random fields. We also assume that the coupling between the electrons and the muon is sufficiently strong for the so-called 'strong collision' model to be valid [9], in which case $G_z(t)$ (corresponding to $\tau_c \to \infty$) in (1) is replaced by a function $g_z(\nu, t)$ defined by

$$g_z(\nu,t) = \mathcal{L}^{-1}\left[\frac{F_z(s+\nu)}{1+\nu F_z(s+\nu)}\right] \qquad F_z(s) = \mathcal{L}[G_z(t)] \tag{2}$$

where $\mathcal{L}$ ($\mathcal{L}^{-1}$) is the Laplace (inverse Laplace) transformation. In the classical motional narrowing limit, $\delta \ll \nu$, $g_z(\nu, t)$ becomes exponential [10]

$$g_z(\nu,t) = \exp\left(-\frac{2\delta^2\tau_c}{1+\omega_0^2\tau_c^2}t\right) \tag{3}$$

where $\tau_c = \nu^{-1}$, and in this case the LF-μSR rate is

$$T_1^{-1} = (2\delta^2\tau_c)/(1+\omega_0^2\tau_c^2). \tag{4}$$

Here, $\omega_0 = eB_{\text{ext}}/m_\mu$ is the muon Larmor frequency. It is the quantity ν we shall now examine, and this is derived from experiment via (4). For three-dimensional systems, considerable theoretical effort has been spent to show that $\nu \sim J^2T^{2K-1}$, where J is a renormalized muon 'bandwidth' (it actually refers to the two-well level splitting for muons at adjacent sites in the system, and no coherent band motion is implied), and K is the renormalized dimensionless coupling between the muon and the electrons. Usually $0.1 \leq K < 0.5$, and for such value of K the assumption that muon hops between different sites are uncorrelated is reasonably good. If we make a similar assumption for a two-dimensional system (an assumption to be readdressed below), then one finds a similar expression for the hopping rate [11]

$$\nu(H,T) = \frac{J_0^2}{D}\exp[-F_{\text{p}}(H,T)]\frac{\cos\pi K(H,T)}{\pi K(H,T)}\left(\frac{T}{D}\right)^{2K-1} \tag{5}$$

where D is an upper energy cutoff. This is similar to three dimensions, except that both the renormalization $\exp(-F_{\text{p}})$ (renormalizing J_0 to J) and the dissipative coupling K depend on the magnetic field—in the case of K, very strongly so. This comes from Landau level quantization effects. Incorporating electron–electron interactions into the theory, one finds [11,12]

$$F_{pl}(t) = 2\sum_q \frac{|u_q|^2}{v_q}\frac{\pi}{\omega_{\text{p}}(q)^2\{1-\exp[-\beta\omega_{\text{p}}(q)]\}}\left(\frac{\partial\varepsilon_q^{\text{R}}(\omega)}{\partial\omega}\right)^{-1}_{\omega=\omega_{\text{p}}(q)} \tag{6}$$

where $\epsilon_q^{\text{R}}(\omega)$ is the real part of the electronic dielectric function and $\omega_{\text{p}}(q)$ is the plasma frequency, whilst u_q and v_q are the muon–electron and electron–electron interactions. Evaluation of K leads to rather complicated expressions, namely

$$K \equiv \sum_q\left|\frac{u_q}{\varepsilon_q}\right|^2\sum_\sigma\int \mathrm{d}\epsilon\,\frac{\partial f_\sigma(\epsilon)}{\partial\epsilon}\sum_{n,m}C_{nm}(q)\,\mathrm{Im}\,[G_{n\sigma}^+(\epsilon)]\,\mathrm{Im}\,[G_{m\sigma}^-(\epsilon)] \tag{7}$$

$$C_{nm}(q) = \frac{2eH}{\pi hc}\frac{n!}{m!}\left[\frac{q^2}{2eH}\right]^{m-n}\exp\left(\frac{-q^2}{2eH}\right)\left[L_n^{m-n}\left(\frac{q^2}{2eH}\right)\right]^2 \tag{8}$$

where L_n^m are associated Laguerre polynomials, $f_\sigma(\epsilon) = [\exp((\epsilon - \mu_\sigma)/T) + 1]^{-1}$, where μ_σ is determined self-consistently for each spin sub-system [13], and $G^{\pm}_{n\sigma}(\epsilon)$ are the retarded/advanced (+/−) single particle Green functions. These spin dependent Green functions are strongly affected by electronic scattering off impurities or defects, and we evaluate them in the self-consistent Born approximation [14], where

$$G^{\pm}_{n\sigma}(\epsilon) = \frac{1}{\epsilon - (n+1/2)\hbar\omega_c - \sigma g H - \Sigma^{\pm}_{\sigma}(\epsilon)} \tag{9}$$

$$\Sigma^{\pm}_{\sigma}(\epsilon) = \pm\frac{2\hbar^2\omega_c}{\pi^2\tau}\sum_n G^{\pm}_{n\sigma}(\epsilon) \tag{10}$$

determine them self-consistently for given σ and H. We ignore, at this level of approximation, the field variation of the impurity scattering time τ, nor do we consider weak localization corrections to $K(H,T)$ in equation (7). Evaluation of the sum in (7) then gives

$$\sum_{nm} \cdots = C_{\bar{n}\bar{n}}(q)\frac{\pi^2}{(\hbar\omega_c)^2}\left[\frac{\sinh(2\pi\Gamma_\sigma/\hbar\omega_c)}{\cosh(2\pi\Gamma_\sigma/\hbar\omega_c) - \cos(2\pi(\epsilon - \Delta_\sigma)/\hbar\omega_c - \pi)}\right]^2 \tag{11}$$

where $\bar{n} = \epsilon/\omega_c$, Δ_σ and Γ_σ is the real and imaginary part of the self energy; we assume that $\mu \gg \Delta_\sigma, \Gamma_\sigma$.

Now what is seen in an experiment will depend on whether the muon prefers to move in the metallic plane or between them; if it moves between them, then the muon–electron coupling $|u_q|^2$ in (7) will be replaced by $|\tilde{u}_q|^2 = \mathrm{e}^{-2qz}|u_q|^2$, where z is the distance between the muon and the nearest metallic plane. Incorporating this into (7) and (11), we may now calculate $\nu(H,T)$ and $T_1^{-1}(H,T)$.

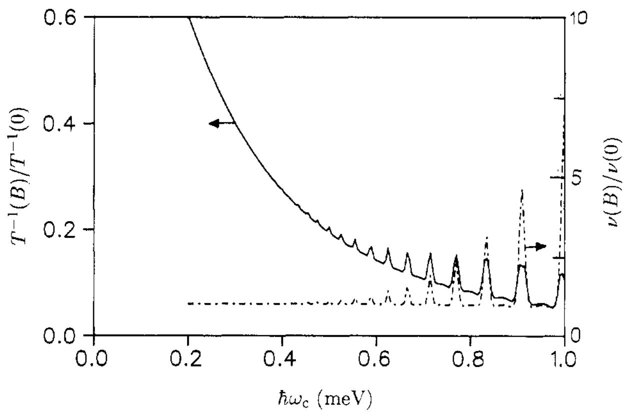

Figure 1. Plots, as functions of normalized magnetic field, of the muon spin relaxation rate T_1^{-1} normalized by its zero-field value (full curve) and the inverse correlation time normalized by its zero-field value (chain curve). Here, $E_F = 10$ meV (corresponding to $q_F \approx 1.35 \times 10^6$ cm^{-1}), $T = 1.5$ K, $\hbar/\tau = 0.012$ meV, $z = 25$ Å and the spin splitting is chosen as $g\sigma H = 0.2\hbar\omega_c$.

In figures 1 and 2, we show these two as a function of field for two different distances from the plane. We assume that the zero-field K is sufficiently large for our assumption

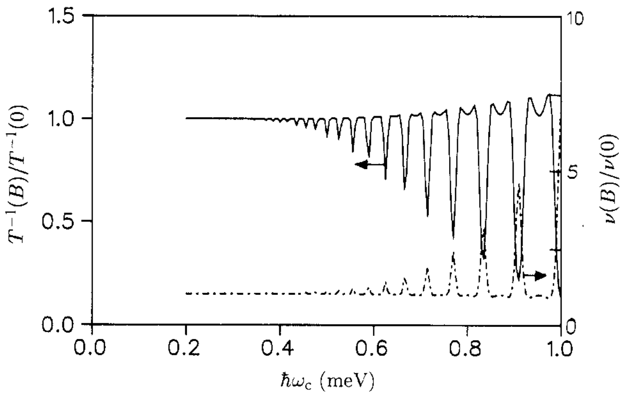

Figure 2. As figure 1, but for $z = 75$ Å.

of incoherent hopping to be valid. Now $\nu(H,T)$ is essentially telling us the local density of states in the region where the muon is diffusing, since $K(H,T)$ measures the low energy dimensionless coupling to the electrons in this region. This behaviour is then directly reflected in T_1, as we see. The ability to measure such local properties is extremely important, since previous transport measurements have only tested the system-averaged effects of impurity scattering on the electronic wavefunctions and density of states.

Examination of equation (4) also reveals another interesting prediction of the theory; for strong muon–electron interactions, such that $\omega_0 \gg \nu(H,T)$, $T_1^{-1} \sim \nu/\omega_0^2$, and then $\nu(H,T)$ and $T_1^{-1}(H,T)$ oscillate in phase. However for much weaker interactions (which will occur for large z, or even, in pure systems around the peaks in $\nu(H,T)$ at low T), $T_1^{-1} \propto \nu^{-1}$, and will exhibit antiphase behaviour (one can work out a whole theory of the resulting non-linearities in the oscillations, from which we refrain here in the absence of experiments with which to compare it). This change in relative phase is illustrated by comparing figures 1 and 2. Figure 2 has a much weaker zero-field coupling K_0, because z is larger.

It is interesting to compare these results with the NMR probe mentioned earlier. They both examine local properties, but muons examine charge correlations, and moreover the phase reversal effects discussed above provide an extra piece of information not available in NMR.

Finally we note that the assumption we have been using throughout, that of incoherent hopping, will break down if the muon is sufficiently far from the planes, or if the system is sufficiently clean to give very large peaks in $\nu(H,T)$. This is because under these conditions the dissipative coupling $K(H,T)$ will be so small that coherence will set in between intersite hopping processes. At this point the muon will effectively '*Bloch delocalize*', and a quite different theoretical treatment is required. This opens up the facinating prospect that one could alternately delocalize and relocalize the muon just by sweeping Landau levels through the Fermi surface [15].

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The great success of the Vagner-Maniv theory [6] leads one to suspect that corrections to the SCBA will also be small here. We have in fact calculated these corrections, for both NMR and LF-μSR, in the framework of weak localization theory. The results will be reported elsewhere.

[15] We have recently shown that Bloch delocalization of muons does take place in insulators; Stamp P C E and Zhang C 1990 *Phys. Rev. Lett.* submitted. Bloch delocalization of muons should also be observable in superconductors below T_c. A detailed theory is in preparation.